Probability & Odds

What are **Odds**?

Odds is the ratio of the number of favorable outcomes of the event to the number of unfavorable outcomes in reduced form.

How do we display the **Odds** for an event?

We can choose to display the **Odds** as $\frac{a}{b}$ or $a : b$.

Example:

A box contains 4 green balls and 6 red balls. What are the odds of randomly drawing a red ball?

Solution:

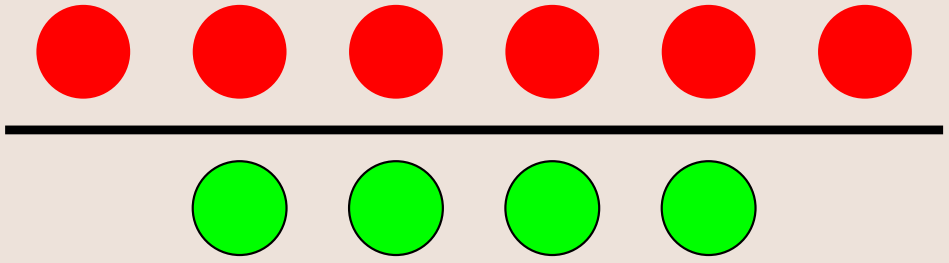
Let's look inside of this box, and get a visual picture of its content.

Since there are 6 favorable outcomes (the number of red balls) to 4 unfavorable outcomes (the number of balls that are not red), so the odds are $\frac{6}{4} = \frac{3}{2}$ or $3:2$.

Now let's look it this problem visually.

Solution Continued:

Since the odds in favor of getting a red ball in the form of a fraction is the number of red balls over the number of not red balls, we get



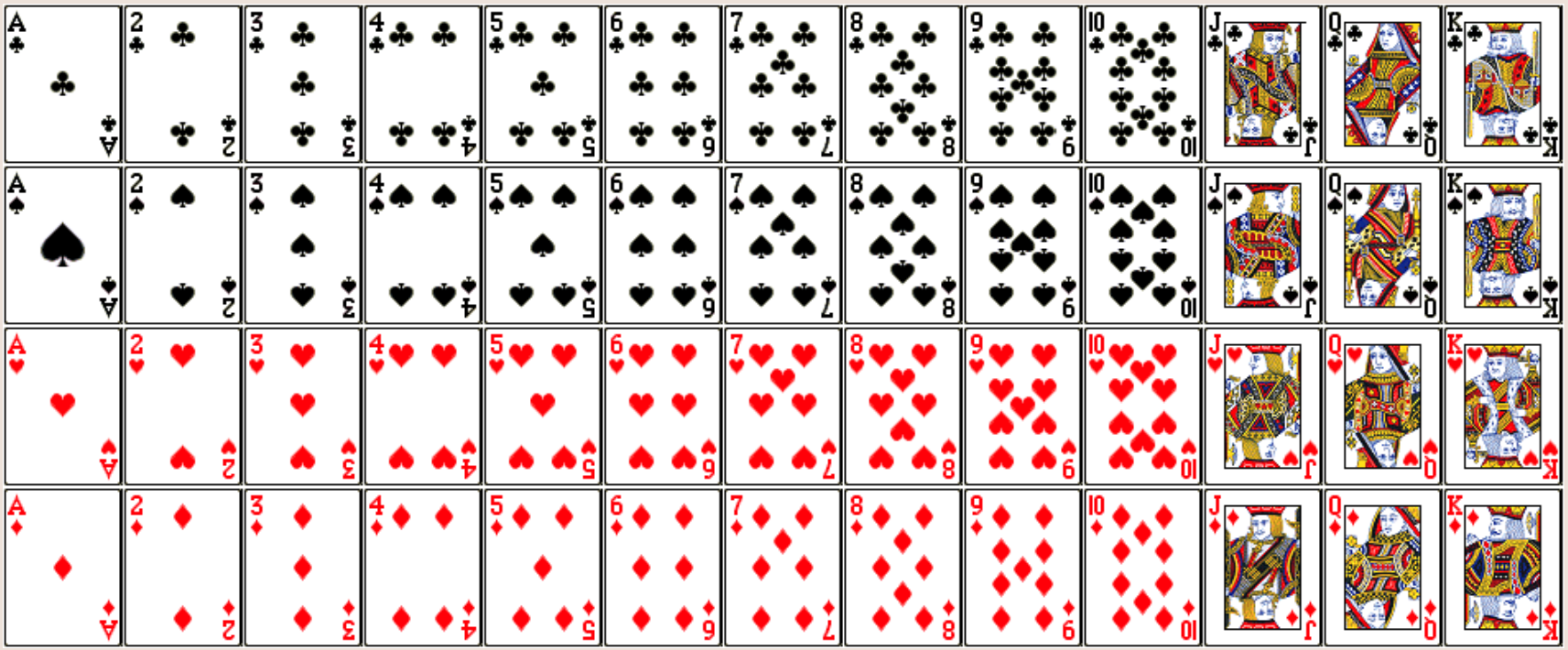
So the odds for drawing a red ball are $\frac{6}{4} = \frac{3}{2}$ or $3 : 2$.

Example:

What are the odds of randomly drawing a face card from an ordinary full deck of playing cards?

Solution:

Lets look at a full deck of playing cards,



Since there are 12 face cards and 40 not face cards, therefore the odds for randomly drawing a face card are $\frac{12}{40} = \frac{3}{10}$ or $3 : 10$.

How do we find **Odds** using **Probability**?

The **Odds** in favor of the event E can be computed by $\dfrac{P(E)}{P(\bar{E})}$.

Example:

Suppose $P(E) = 0.5\%$, what are the odds in favor of the event E?

Solution:

We have $P(E) = 0.5\%$, so the odds are

$$\frac{P(E)}{P(\bar{E})} = \frac{.5\%}{1 - .5\%} = \frac{.5\%}{99.5\%} = \frac{.005}{.995} = \frac{5}{995} = \frac{1}{199} \text{ or } 1 : 199.$$

Example:

Suppose the probability of getting tails when tossing a loaded coin is 0.55. What are the odds of getting tails when tossing this coin?

Solution:

$$\begin{aligned}\text{Odds for getting tails} &= \frac{P(T)}{P(\bar{T})} \\ &= \frac{.55}{1-.55} = \frac{.55}{.45} \\ &= \frac{55}{45} = \frac{11}{9}\end{aligned}$$

So the odds for getting tails when tossing this coin are $11:9$

How do we find **Probability** when **Odds** are given?

If the **Odds** in favor of the event E is $a : b$, then

$$P(E) = \frac{a}{a+b} \quad \& \quad P(\overline{E}) = \frac{b}{a+b}$$

Example:

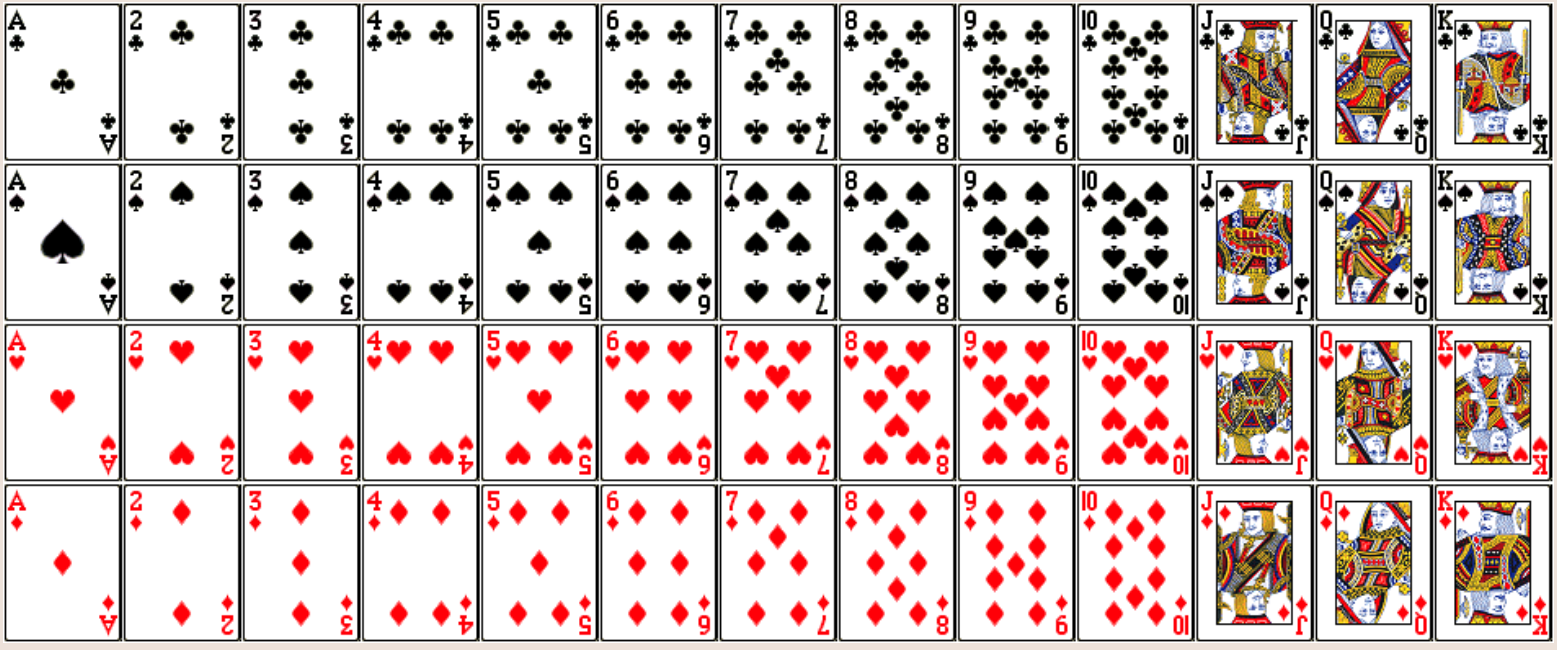
Suppose the odds that the Dallas Cowboys win the super bowl this year are $2 : 11$. What is the probability that they win the super bowl this year?

Solution:

Since the odds are $2 : 11$, therefore $P(\text{ Win }) = \frac{2}{2+11} = \frac{2}{13}$.

Example:

Consider a full-deck of playing cards shown below.



What is the probability of randomly drawing an ace?

What are the odds of randomly drawing an ace?

Solution:

$$\begin{aligned}\text{Probability(Draw an ace)} &= \frac{\text{Number of aces}}{\text{Total number of cards}} \\ &= \frac{4}{52} = \frac{1}{13} \\ &\approx 0.077\end{aligned}$$

$$\begin{aligned}\text{Odds for drawing an ace} &= \frac{\text{Number of ace cards}}{\text{Number of not ace cards}} \\ &= \frac{4}{48} = \frac{1}{12}\end{aligned}$$

So the odds of randomly drawing an ace are $1 : 12$

"I wish we hadn't learned probability
'cause I don't think our odds are good."